Diophantine Equations 1.

S1.1. Mathematical national competition in Croatia 2002: 1st grade 3/4.

Determine all triplets (x, y, z) of positive integers which are solutions of the equation

$2x^2y^2 + 2y^2z^2 + 2z^2x^2 - x^4 - y^4 - z^4 = 576.$

Hint: Factorize the expression on the left-hand side.

S1.2. Mathematical competition in Lithuania October 2001

Find the integer solutions of the equation $x - y = x^2 + xy + y^2$.

S1.3. Estonian Mathematical Olympiad 1999 Final Round 11th Grade 1/5.

Find all pairs of integers (m, n) such that $(m-n)^2 = \frac{4mn}{m+n-1}$.

S1.4. Math Olympiad in Slovenia 1998 First Round Grade Four 1/4.

Show that there are no positive integers k, m, n satisfying the equation $n^2 + m^3 = m^k$.

S1.5. Belorussian Math. Competition, Minsk, 2002, 11th class

Let p be prime. Find the nonnegative integers m, n such that $n^2 - 5p^m = 1$.

Diophantine Equations 2.

S2.1. Math Olympiad in Slovenia 1998 First Round Grade Two 1/4.

Show that the equation $x^2 + y^2 + z^2 = (xy)^2$ has no positive integer solutions.

S2.2. Mathematical competition in Lithuania October 2001

Find all positive integer values which can be written as the ratio of the product and the sum of two distinct positive integers.

S2.3. Czech and Slovak Mathematical Olympiad April 2002: Final Round

Solve the system

$(4x)_5 + 7y = 14,$
$(2y)_5 - (3y)_7 = 74,$

in the domain of the integers, where $(n)_k$ stands for the multiple of the number k closest to the number n.

S2.4. Estonian Mathematical Olympiad 1999 Final Round 10th Grade 1/5.

Find all pairs of integers (a, b) such that $a^2 + b = b^{1999}$.

S2.5. British Mathematical Olympiad December 2001: Round 1 (Time: 3.5 hours) 3/5.

Find all positive real solutions to the equation

$$x + \left[\frac{x}{6}\right] = \left[\frac{x}{2}\right] + \left[\frac{2x}{3}\right]$$

where [t] denotes the largest integer less than or equal to the real number t.

Diophantine Equations 3.

S3.1. Mathematical Competition 1997 Lithuania

Can the number 500…02 be the sum of cubes of the some consecutive integers?

S3.2. Math Olympiad in Slovenia 1998 Final Round Grade Two 2/4.

Find all pairs (p, q) of real numbers such that p + q = 1998 and that the equation $x^2 + px + q = 0$ has only integer solutions.

S3.3. Japanese Mathematical Olympiad 2002 First Round

Find the number of all pairs of integers (x, y) such that $xy^2 + xy + x^2 – 2y – 1 = 0$.

S3.4. Estonian Open Contest 1999 (9th and 10th grade)

Prove that the value of the expression $1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+n}}}$ is not an integer for any integer n.

S3.5. Proposed at 1995 IMO

Solve the equation $x + y^2 + z^3 = xyz$ where x, y and z are positive integers such that z is the greatest common divisor of x and y.

Diophantine Equations 4.

S4.1. Math Olympiad in Slovenia 1998 Final Round Grade Three 4/4.

Alf was going to eight-year elementary school. At the end of every schoolyear he showed the certificate to his father. If he was promoted, his father multiplicated Alf’s age with the grade Alf just passed and gave him as many cats as the multiplication showed. During the schooling

Alf was kept back once. When he finished elementary school he found out that the number of cats he had got from his father is divisible by 1998. Which grade did Alf repeat?

S4.2. Serbian Mathematical Olympiad 1999 1st form (Time: 4 hours) 3/5.

Let an integer m be given. Prove that there exist at least one pair (x, y) of integers, such that $2x^2 + 11xy + 12y^2 + 4x + 5y + 6 = 2m$.

S4.3. Spanish Mathematical Olympiad 1999 First Local Round First Day 3/3.

Determine todos los números naturales n para los que el número $2^n + 2^{1999}$ es un cuadrado perfecto (esto es, el cuadrado de un número natural).

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Find all natural numbers n for which the number $2^n + 2^{1999}$ is a perfect square.

S4.4. Mathematical Competition 1997 Lithuania

Find at least one solution of the equation $x^3 + y^5 = z^4$ in positive integers. Prove that there exist infinitely many solutions of the kind.

S4.5. Mathematical Olympiads' Correspondence Program 1996 Canada

Find all integer triplets (x, y, z) that satisfy the system $3 = x + y + z = x^3 + y^3 + z^3$.

Diophantine Equations 5.

S5.1. Math Olympiad in Slovenia 2002 First Round Grade Three 1/4.

Find all pairs of positive integers m and n such that $1\cdot2\cdot3\cdot\ldots\cdot(m-1)\cdot m + 3 = n^2$ holds.

S5.2. Spanish Mathematical Olympiad 1999 Second Local Round Second Day 1/3.

Halla todos los pares de números naturales x, y (x < y) tales que la suma de todos los números naturales comprendidos estrictamente entre ambos es igual a 1999.

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Find all pairs of natural numbers x, y (x < y) such that the sum of the natural numbers which are strictly between both numbers is 1999.

S5.3. First Round of the Selection Test for IMO 1990 Japan

Determine the greatest integer n such that $4^{27} + 4^{500} + 4^n$ is the square of an integer.

S5.4. Austrian Mathematical Olympiad (Final Round, qualifying day): May 2002 1/4.

Determine all integers a and b such that $(19a + b)^{18} + (a + b)^{18} + (19b + a)^{18}$ is a perfect square.

S5.5. German National Mathematics Competition 1998 Second Round

Determine all integer solutions (x, y, z) of the equation $xy + yz + zx - xyz = 2$.

Diophantine Equations 6.

S6.1. South Africa, Stellenbosch Camp December 1998: Test 1 (Time: 3 hours) 3/5.

Two of the altitudes of a triangle are of length 9 cm and 29 cm. The third altitude is also a whole number of centimetres in length. What are the possible lengths of the third altitude?

S6.2. Math Olympiad in Slovenia 2002 Final Round Grade Three 1/4.

Let k be a positive integer such that the quadratic equation $kx^2 - (1 - 2k)x + k - 2 = 0$ has rational solutions. Prove that k is the product of two consecutive integers.

S6.3. Spanish Mathematical Olympiad 1999 National Round First Day 2/3.

Probar que existe una sucesión de enteros positivos $a_1, a_2, \dots, a_n, \dots$ tal que $a_1^2 + a_2^2 + \dots + a_n^2$ es un cuadrado perfecto para todo entero positivo n.

***

Prove that there exists a sequence of positive integer $a_1, a_2, \dots, a_n, \dots$ such that $a_1^2 + a_2^2 + \dots + a_n^2$ is a perfect square for every positive integer n.

S6.4. Russian Mathematical Olympiad 2002 IV-th (District) round 11-th form 1/8.

Real numbers x and y are such that for any two distinct odd prime numbers p and q the number $x^p + y^q$ is rational. Show that x and y are rational.

S6.5. Austrian Mathematical Olympiad May 2001: National Competition (Day 1) 1/3.

Prove that $\frac{1}{25}\sum_{k=0}^{2001}\left[\frac{2^k}{25}\right]$ is an integer. ([x] denotes the largest integer less than or equal to x.)

Diophantine Equations with Figures 1.

S7.1. Sharp Calculator Competition January 2001

A certain positive integer n has all its digits equal to 3 and is exactly divisible by 383. What are the last five digits of $\frac{n}{383}$?

S7.2. Math Olympiad in Slovenia 1998 First Round Grade Three 1/4.

Let b > 1 and n > 1 be positive integers such that the number $\overbrace{11...1}^{n \text{ digits}}{}_{(b)}$, read in base b, is a prime. Prove that n is a prime, too.

S7.3. Japanese Mathematical Olympiad 2002 First Round

Find all possible m such that $(m-2)^2$ is a 3-digit number abc, while m^2-1 is the 3-digit number cba. (Here a, b, c are integers, $0 < a, c \le 9$, $0 \le b \le 9$.) Find all such possible m*'s*.

S7.4. Estonian Open Contest 1999 (9^{th} and 10^{th} grade)

Find all four-digit numbers n such that multiplying n by $\frac{9}{2}$ we obtain the number which is composed of the same digits as n but in the opposite order.

S7.5. Spanish Mathematical Olympiad 2002 Final round 1/6.

Sea n un número natural y m el que resulta al escribir en orden inverso las cifras de n. Determinar, si existen, los números de tres cifras que cumplen 2m + S = n, siendo S la suma de las cifras de n.

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For a positive integer n, let m be the number obtained by reversing the order of the digits of n. Is there any n 3-digits number such that 2m + S = n, where S is the sum of the digits of n?

Diophantine Equations with Figures 2.

S8.1. Estonian Spring Open Contest: February 2002: Seniors 4/5.

Call a 10-digit natural number magic if it consists of 10 distinct digits and is divisible by 99999. How many such magic numbers are there (not starting with digit 0)?

S8.2. South Africa, Potchefstroom Camp July 2001: Test 1 (Time: 4 h 15 m) 2/5.

Find those 6 digit numbers that are equal to the square of the number formed by their last 3 digits.

S8.3. Math Olympiad in Slovenia 1998 Final Round Grade Four 1/4.

Let n be a positive integer. If the number 1998 is written in base n, we get a three-digit number and the sum of the three digits is 24. Determine all possible values of n.

S8.4. Mathematical Competition 1990 New Zealand

Find the digit m if the number 88…88m99…99 is divisible by 7, where each 8 and each 9 occurs 50 times.

S8.5. Mathematical Olympiads’ Correspondence Program 1996 Canada

Denote by s(n) the sum of the base-10 digits of the natural number n. The function f(n) is defined on the natural numbers by $f(0) = 0$, $f(n) = f(n - s(n)) + 1$ ($n = 1, 2, \ldots$). Prove, or disprove, that $f(m) \leq f(n)$ whenever $1 \leq m \leq n$.

Diophantine Equations with Figures 3.

S9.1. Estonian Mathematical Olympiad 2002 Final Round 9th Grade 2/5.

Do there exist distinct non-zero digits *a*, *b* and *c* such that the two-digit number $\overline{ab}$ is divisible by *c*, the number $\overline{bc}$ is divisible by *a* and $\overline{ac}$ is divisible by *b*?

S9.2. Austrian Mathematical Olympiad April 2001: Regional Competition 1/3.

Let n be an integer and $S(n) = \sum_{k=0}^{2000} n^k$. Determine the last digit (i.e. the ones-digit) in the decimal expansion of S(n).

S9.3. Math Olympiad in Slovenia 2002 Final Round Grade One 1/4.

The number 38 is the smallest positive integer whose square ends with three fours ($38^2 = 1444$). Which is the next positive integer having this property?

S9.4. Mathematical Competition 1997 Lithuania

Prove that for any positive integer m there always exists a positive integer n such that its decimal expression does not contain zeros and the sum of its digits is equal to the sum of digits of the number mn.

S9.5. South Africa, Stellenbosch Camp December 1998: Test 1 (Time: 3 hours) 4/5.

Prove that, if 5^n begins with 1, then 2^{n+1} also begins with 1, where n is a positiv integer. Is the converse true?

Diophantine Equations with Figures 4.

S10.1. Japanese Mathematical Olympiad 1991 First Round

678
Let $A = 99...9$. Determine the sum of the all digits of A^2 in the decimal system.

S10.2. Flanders Mathematics Olympiad 1999 Final Round 1/4.

Determine all natural numbers, having 6 digits, say abcdef, with $a \neq 0$ and $d \neq 0$ and such that $abcdef = (def)^2$.

S10.3. Estonian Mathematical Olympiad 2002 Final Round 12th Grade 2/5.

Does there exist an integer containing only digits 2 and 0, which is a k-th power of a positive integer with $k \geq 2$?

S10.4. Math Olympiad in Slovenia 2002 First Round Grade Two 3/4.

Find three consecutive odd numbers a, b and c, such that $a^2 + b^2 + c^2$ is a four-digit number with all digits equal to each other.

S10.5. Irish Mathematical Olympiad 1998: First Day (Time: 3 hours) 3/5.

Show that no integer of the form xyxy in base 10 (where x and y are digits) can be cube of an integer.
Find the smallest base $b > 1$ for which there is a perfect cube of the form xyxy in base b.

Number Theory 1.

S11.1. Albanian Mathematical Olympiad March 2002 9th class 1/5.

The natural numbers m and n satisfy the condition $\frac{m^2 + n^2}{mn} \in N$. Prove that m = n.

S11.2. British Mathematical Olympiad 1998 Round 1 2/5.

Let $a_1 = 19$, $a_2 = 98$. For $n \geq 1$ define a_{n+2} to be the remainder when $a_n + a_{n+1}$ is divided by 100. What is the remainder when $a_1^2 + a_2^2 + ... + a_{1998}^2$ is divided by 8?

S11.3. Lithuanian Mathematical Olympiad 1998

For which primes p the number $\frac{2^{p-1} - 1}{p}$ is the square of an integer?

S11.4. Spanish Mathematical Olympiad 2002 Final round 4/6.

La función g se define sobre los números naturales y satisface las condiciones:
$g(2) = 1$;
$g(2n) = g(n)$;
$g(2n + 1) = g(2n) + 1$.
Sea n un número natural tal que $1 \leq n \leq 2002$. Calcula también cuántos valores de n satisfacen $g(n) = M$.

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Let g be a function on natural numbers satisfying the following conditions
$g(2) = 1$;
$g(2n) = g(n)$;

$g(2n + 1) = g(2n) + 1$.
Let n be a natural number such that $1 \le n \le 2002$. What is the maximum value M of g(n)? For how many values of n does g(n) = M hold?

S11.5. Estonian Mathematical Olympiad 1999 Final Round 12th Grade 1/5.

Let a, b, c and d be non-negative integers. Prove that the numbers 2^a7^b and 2^c7^d give the same remainder when divided by 15 iff the numbers 3^a5^b and 3^c5^d give the same remainder when divided by 16.

Number Theory 2.

S12.1. Manitoba Mathematical Contest, February 2001 Canada

a, b, c, d are distinct integers such that $(x – a)(x – b)(x – c)(x – d) = 4$ has an integral root r. Prove that $a + b + c + d = 4r$.

S12.2. Macedonian Mathematical Competition 2002 II Round I Class 1/4.

Let a and b integers such that $(16a + 17b)(17a + 16b)$ is divisible by 11. Prove that it is also divisible by 121.

S12.3. Math Olympiad in Slovenia 1998 First Round Grade Three 2/4.

The quadratic function $p(x) = x^2 + ax + b$ with arbitrary integers a and b is given. Show that for every integer n there exists an integer m such that $p(n)p(n + 1) = p(m)$.

S12.4. Estonian Open Contest 1999 (11th and 12th grade)

Let a be an integer, whose square when divided by n gives the remainder 1. What is the remainder when the number a is divided by n, if
a) n = 16;
b) $n = 3^k$, where k is a positive integer?

S12.5. South Africa, Rhodes Camp April 2001: Test 2 (Time: 4.5 hours) 3/3.

Let $S = \{0, 1, 2, \dots , 1994\}$. Let a and b be two positive numbers in S which are relatively prime. Prove that the elements of S can be arranged into a sequence $s_1, s_2, s_3, \dots , s_{1995}$ such that $s_{i+1} – s_i \equiv \pm a$ or $\pm b \pmod{1995}$ for $i = 1, 2, \dots , 1994$.

Number Theory 3.

S13.1. Estonian Autumn Open Contest: October 2001: Juniors 2/5.

Find the remainder modulo 13 of the sum $1^{2001} + 2^{2001} + 3^{2001} + \dots + 2001^{2001}$.

S13.2. British Maths Olympiad 2000 Round 2

Find positive integers a and b such that $\left(\sqrt[3]{a}+\sqrt[3]{b}-1\right)^2 = 49+20\sqrt[3]{6}$.

S13.3. Albanian Mathematical Olympiad March 2002 11th class 4/5.

Prove that if p is an odd prime, than

a) $p \mid C_p^k \quad \forall k = 1, 2, ..., p-1$,

b) p divides the number $\left[\left(2+\sqrt{5}\right)^p\right] - 2^{p+1}$.

([x] is the greatest integer, not greater than x.)

S13.4. Iranian Mathematical Olympiad 2002 First Round 1/6. Time: 2x4 hours

Let p and n be natural numbers such that p is a prime and 1 + np is a perfect square. Prove that n + 1 is a sum of p perfect squares.

S13.5. Austrian Mathematical Olympiad (Final Round, qualifying day): May 2002 3/4.

Let $f(x) = \dfrac{9^x}{9^x+3}$. Calculate the sum of all expressions of the form $f\left(\dfrac{k}{2002}\right)$ with k being an integer between 0 and 2002 for which the fraction $\dfrac{k}{2002}$ is in lowest terms.

Number Theory 4.

S14.1. Math Olympiad in Slovenia 2002 Final Round Grade One 2/4.

Tom collected stamps. He took his savings and he found out that he had 2002 cents. He decided to spend all this money to buy stamps. His friend offered him some little stamps at 10 cents each and some greater stamps at 28 cents each. Tom decided to buy as many stamps as he could. How many stamps has he bought?

S14.2. Mathematical competition in Lithuania October 2001

Let $a_n = \sqrt{\left|60\sqrt{11}-199\right|} + \sqrt{60\sqrt{11}+171+n}$. Is there any positive integer in the sequence a_1, a_2, … a_n?

S14.3. Albanian Mathematical Olympiad March 2002 12th class 2/5.

The natural number a is such that $a^2 \equiv 1 \pmod{n}$. Find a (mod n) if
$n = p^k$, where p is an odd prime and k is a natural number.

S14.4. Hellenic Mathematical Olympiad February 2002 4/4.

a) For the positive integers p, q, r, a is given that $pq = ra^2$, where r is prime and p, q are relatively prime. Prove that one of the numbers p, q is a perfect square of a positive integer.

b) Examine if the exists a prime positive integer p such that $p(2^{p+1} - 1)$ is a perfect square of a positive integer.

S14.5. Irish Mathematical Olympiad 1998: First Day (Time: 3 hours) 5/5.

If x is a real number such that $x^2 - x$ is an integer, and, for some $n \geq 3$, $x^n - x$ is also an integer, prove that x is an integer.

Number Theory 5. – Prime Numbers

S15.1. Mathematical Competition 1997 Lithuania

Is there a prime number which is equal to the difference of the fifth powers of some two prime numbers?

S15.2. Flanders Mathematics Olympiad 1999 Final Round 4/4.

Consider $a, b, m, n \in N \setminus \{0, 1\}$ such that $a^n - 1$ and $b^m + 1$ are prime numbers. Give as much information as possible concerning the integers a, b, n and m.

S15.3. Irish Mathematical Olympiad May 2002: Test 2 (Time: 3 hours) 2/5.

Suppose n is a product of four distinct primes a, b, c, d such that

(a) $a + c = d$;
(b) $a(a + b + c + d) = c(d - b)$;
(c) $1 + bc + d = bd$.

Determine n.

S15.4. Thai Mathematical Olympiad 2001

Let a, b, c be positive integers such that $(a + 1)(b + 1)(c + 1) = 3abc$, a is a prime number less than 5 and $b + c > 6$. Find $a^2 + b + c$.

S15.5. Italian Mathematical Olympiad May 2002 5/6.

Prove that if $m = 5^n + 3^n + 1$ is prime, then 12 divides n.

Number Theory – Canonical Form 1.

S16.1. Japanese Mathematical Olympiad 1998 First Round

Find the smallest positive integer such that all the digits of its decimal representation are equal and it is divisible by 1998.

S16.2. Flanders Mathematics Olympiad 1998 Final Round 1/4.

Proof that there exist integers a, b and c satisfying $0 < a < b \le c < 2a$ and $a + b + c = 1998$, and such that gcd(a, b, c) is maximal. Determine such a triple. Is this triple unique?

S16.3. Austrian Mathematical Olympiad April 2002 Qualifying Round 1/4.

Determine the smallest positive integer x such that each of the following fractions are in lowest terms:

$$\frac{3x+9}{8}, \frac{3x+10}{9}, \frac{3x+11}{10}, \ldots, \frac{3x+49}{48}.$$

S16.4. Estonian Autumn Open Contest: October 2001: Seniors 3/5.

For any positive integer n, denote by S(n) the sum of its positive divisors (including 1 and n).
a) Prove that $S(6n) \le 12S(n)$ for any n.
b) For which n does the equality $S(6n) = 12S(n)$ hold?

S16.5. Singapore Math Olympiad 1997

Find all positive integers n such that:

a) n has exactly 6 positive divisors: 1, d_1, d_2, d_3, d_4, n,
b) $1 + n = 5(d_1 + d_2 + d_3 + d_4)$ holds.

Number Theory – Canonical Form 2.

S17.1. Japanese Mathematical Olympiad 1998 First Round

How many positive integers n are there not exceeding 1998 such that $n^{1998} - 1$ is divisible by 10?

S17.2. Mathematical Competition 1997 Lithuania

Find the least integer n with the property that the fractions $\frac{7}{n+9}, \frac{8}{n+10}, \frac{9}{n+11}, \ldots, \frac{31}{n+33}$ are irreducible.

S17.3. Estonian Mathematical Contest 1998 Final round

Let d_1 and d_2 be positive divisors of the positive integer n and let $\gcd\left(\frac{n}{d_1}, d_2\right) = \gcd\left(\frac{n}{d_2}, d_1\right)$. Prove that $d_1 = d_2$.

S17.4. Czech and Slovak Mathematical Olympiad January 2002: Second Round

Find all pairs of natural numbers x and y for which $x^2 = 4y + 3\cdot[x, y]$, when [x, y] denotes the least common multiple of the numbers x and y.

S17.5. British Mathematical Olympiad 1998 Round 2 3/4.

Suppose x, y, z are positive integers satisfying the equation $\frac{1}{x} - \frac{1}{y} = \frac{1}{z}$ and let h be the highest common factor of x, y, z. Prove that hxyz is a perfect square. Prove also that h(y – x) is a perfect square.

Number Theory – Canonical Form 3.

S18.1. Math Olympiad in Slovenia 2002 Final Round Grade Two 2/4.

Find the smallest positive integer that can be written as a sum of 9, 10 and 11 consecutive positive integers.

S18.2. Estonian Mathematical Olympiad 2002 Final Round 10th Grade 1/5.

The greatest common divisor d and the least common multiple v of positive integers m and n satisfy the equality 3m + n = 3v + d. Prove that m is divisible by n.

S18.3. Serbian Mathematical Olympiad 1999 1st form (Time: 4 hours) 5/5.

A natural number $n \geq 2$ is divided by all natural numbers smaller than n and the remainders are recorded. Find those natural numbers n for which the distinct remainders is equal to n.

S18.4. Japanese Mathematical Olympiad 1992 First Round

Let A be the set of all positive integers of the form $2^{n_1} \cdot 3^{n_2} \cdot 5^{n_3} \cdot 7^{n_4} \cdot 11^{n_5} \cdot 13^{n_6}$ where $0 \leq n_i \leq 1$ for $1 \leq i \leq 6$. Determine $\sum_{a \in A} \frac{1}{a}$.

S18.5. Irish Mathematical Olympiad 1998: Second Day (Time: 3 hours) 1/5.

Find those positive integers n that have exactly 16 positive integral divisors $d_1, d_2, \ldots, d_{16}$ such that $1 = d_1 < d_2 < \ldots < d_{16} = n$, $d_6 = 18$ and $d_9 - d_8 = 17$.

Existence and construction in Number Theory 1.

S19.1. Canadian Mathematical Competition 2002

Trouver toutes les valeurs de n telles que 1! + 2! + 3! + … + n! soit un carré parfait.

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Find all values of n such that 1! + 2! + 3! + … + n! is a perfect square.

S19.2. Estonian Mathematical Olympiad 1999 Final Round 12th Grade 5/5.

The numbers 0, 1, 2, … , 9 are written (in some order) on the circumference of a circle. Prove that
a) there are three consecutive numbers whose sum is at least 15;
b) it is not necessarily the case that there exist three consecutive numbers with the sum more than 15.

S19.3. Hellenic Mathematical Olympiad February 2002 (Juniors) 3/4.

Determine all positive integers x, y, z with $x \le y \le z$ satisfying xy + yz + zx – xyz = 2.

S19.4. Thai Mathematical Olympiad 2001

How many three digit positive numbers are there such that their reciprocals can be represented by the sum of two reciprocals of an even and an odd positive integer?

S19.5. Pan-African Mathematics Olympiad July 2001: Day 1 (Time: 4.5 hours) 1/3.

Find all integers n ≥ 1 such that $\frac{n^3+3}{n^2+7}$ is an integer.

Existence and construction in Number Theory 2.

S20.1. Macedonian Mathematical Competition 2002 I Round I Class

The sum of 20 natural numbers is 2002. Find the maximum value of their greatest common divisor.

S20.2. Estonian Spring Open Contest: February 2002: Juniors 1/5.

Is it possible to arrange the integers 1 to 16
a) on a straight line;
b) on a circle
so that the sum of any two adjacent numbers is the square of an integer?

S20.3. Mathematical competition in Lithuania October 2001

Find all possible integers m such that the expression $\sqrt{m^2+m+1}$ is an integer too.

S20.4. Irish Mathematical Olympiad May 2002: Test 1 (Time: 3 hours) 3/5.

Find all triples of positive intgers (p, q, n), with p and q primes, satisfying

p(p + 3) + q(q + 3) = n(n + 3).

S20.5. Bulgarian Mathematical Competition March 2001

Find the smallest integer n, for which n > 1 and $\frac{1^2+2^2+...+n^2}{n}$ is a perfect square.

Existence and construction in Number Theory 3.

S21.1. First Round of the Selection Test for IMO 1990 Japan

Find the minimum positive integer n such that the trailing three digits of n^2 are the same number but not 0 in the decimal system.

S21.2. Auckland Mathematical Olympiad 1998: Division 2

Find all prime numbers p for which the number $p^2 + 11$ has exactly 6 different divisors (including 1 and the number itself).

S21.3. Greek Mathematical Olympiad 2000 2/4.

Find the prime number p so that $1 + p^2 + p^3 + p^4$ is a perfect square, i.e. the square of an integer.

S21.4. South Africa, Rhodes Camp April 2001: Test 3 (Time: 4.5 hours) 3/3.

Let $m \geq 2$ be an integer. Find the smallest integer $n > m$ such that for any partition of the set $\{m, m + 1, \dots , n\}$ into two subsets, at least one subset contains three numbers a, b, c (not necessarily different) such that $a^b = c$.

S21.5. Estonian Mathematical Contest 1998 Final round

Find all the integers $n > 2$ such that $(2n)! = (n - 2)!\cdot n!\cdot (n + 2)!$.

Geometric Number Theory

S22.1. Mathematical county competition in Croatia 2002: 3rd grade 3/4.

Determine the lengths of the edges of a regular quadrangular prism if it is known that they are all integer and that the area of lateral surfaces numerically equals the sum of the lengths of all its edges.

S22.2. Math Olympiad in Slovenia 2002 First Round Grade Three 2/4.

A cube with integer-valued volume and a square with integer-valued area are given. The edge of the cube is 1 unit longer than the side of the square. Prove that the edge of the cube and the side of the square have integer-valued lengths.

S22.3. South Africa, Stellenbosch Camp December 2000: Test 2 (Time: 3.5 hours) 5/7.

Triangle ABC is non-obtuse, with sides of integer lengths, and $a < b < c$. A perpendicular is dropped from B to meet AC in D. Prove that $AD - CD = 4 \Leftrightarrow AB - BC = 2$.

S22.4. Irish Mathematical Olympiad May 2002: Test 2 (Time: 3 hours) 5/5.

Let ABC be a triangle whose side lengths are all integers, and let D and E be the points at which the incircle of ABC touches BC and AC respectively. If $\left|AD^2 - BE^2\right| \leq 2$, show that AC = BC.

S22.5. Mathematical Competition 1991 New Zealand

In triangle ABC, angle A is twice angle B, angle C is obtuse, and the three side lengths a, b, c are integers. Determine the smallest possible perimeter of the triangle.